SLR-MM – 501

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – I)
Statistical Computing (New CBCS)

Day and Date : Monday, 16-11-2015 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Attempt* ***five*** *questions.*
2) Q. No. ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No.* ***3*** *to Q. No.* ***7****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative : **5**

1) The minitab command to open saved worksheet is __________

a) OPEN b) LOAD c) EXEC d) RUN

2) Let > X ← 10; then

> X < 100 && X > 5

results __________

a) TRUE b) FALSE c) NA d) None of these

3) __________ is user defined data type.

a) structure b) union
c) class d) enumeration

4) In boot-strap technique __________ method is used for resampling.

a) stratified b) systematic
c) SRSWR d) SRSWOR

5) If X and Y are independent standard normal variates then $\frac{Y}{X}$ is __________ variate.

a) normal b) t
c) chi-square d) cauchy

P.T.O.

B) Fill in the blanks : **5**

1) R command for obtaining descriptive statistic of set of observations in X is _________

2) In minitab __________ command is used to put data row wise in worksheet.

3) Let U is U(0, 1) variate then __________ is geometric variate with parameter p.

4) Using CLT, the number of U(0, 1) variates required to obtain single N(0, 1) variate is __________

5) A ________ can have both variables and functions as members.

C) State whether the following statements are **true** or **false** : **4**

1) We can have different parts of current minitab projects.

2) Text files cannot be imported in R-software.

3) Void is not built-in data type.

4) If X is geometric variate the X + 1 is waiting time variate.

2. a) Answer the following : **6**

i) What are the pre-defined data types in C++ ?

ii) Explain any three commands in R-language.

b) Write short notes on the following : **8**

i) Tests for uniformity of random numbers.

ii) Random sample generation from mixture of distributions.

3. a) Discuss the congruential method of generating uniform variates.

b) State and prove the result to generate observations from binomial distribution with specified parameters n and p. Also write algorithm for the same. **(6+8)**

4. a) Discuss Jack-knife technique with its limitations.

   b) Let $X_1, X_2, ..., X_n$ be iid Bernoulli random variables with probability of success p. Obtain Jack-knife estimator for p^2. **(6+8)**

5. a) What are control structures and loops in C++ ? Explain their merits and demerits.

   b) Write C++ program to generate an observation from exponential distribution with mean 4. **(7+7)**

6. a) Describe the procedure to test independence of attributes in 2×2 contingency tables in MS-Excel.

   b) Describe the use of following minitab commands with suitable example.

      i) READ

      ii) SET

      iii) INSERT. **(8+6)**

7. a) What is Box-Muller transformation ? Explain how it is used to generate random variates from N(0, 1) distribution.

   b) Describe a procedure of generating a random vector from a bivariate exponential distribution. Also write algorithm for the same. **(7+7)**

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SLR-MM – 502

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – II)
Real Analysis (New CBCS)

Day and Date : Wednesday, 18-11-2015 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **Compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) The finite intersection of open sets is __________
   a) An open set b) A closed set
   c) Both open and closed d) Neither open nor closed

2) Subset of a countable set is __________
   a) Always countable b) Always uncountable
   c) May or may not be countable d) None of these

3) The collection of all the limit points of a set is called its __________
   a) Interior set b) Derival set
   c) Neighbourhood d) None of these

4) The function $f(x) = |x|$ is __________
   a) Step function b) Continuous
   c) Discontinuous at zero d) None of these

5) Every Cauchy sequence is a __________ sequence.
   a) Divergent b) Convergent
   c) Monotonic d) Oscillatory

B) Fill in the blanks : **5**

1) A set is closed if it includes all of its __________ points.
2) Least upper bound of a set is also called as __________
3) For an open set, every point of the set is its ___________ point.
4) Countable union of countable sets is ___________
5) Finite union of closed sets is always ___________

C) State whether the following statements are **True** or **False** : **4**

1) Root test can be applied for any series to check its convergence.
2) If exists, infimum is always unique.
3) Arbitrary union of closed sets is always closed.
4) Set of integers is a countable set.

2. a) State the following :
   i) Cauchy criterion of convergence of a series.
   ii) Bolzano-Weistrauss theorem
   iii) Heine-Borel theorem.

   b) Write short note on the following :
   i) Mean value theorem.
   ii) Limit superior of a sequence. **(6+8)**

3. a) Prove that a set is open iff its compliment is closed.
   b) Prove or disprove : Arbitrary union of open sets is open.
   c) Show that the set of rationals is a countable set. **(5+5+4)**

4. a) Prove or disprove : Monotonic bounded sequence always converges.
   b) Examine the convergence of following sequences :

   i) $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots \frac{1}{n!}$ for all $n \in N$

   ii) $S_n = n^{1/n}$ for all $n \in N$. **(8+6)**

5. a) Describe comparison test and ratio test of convergence of a series.

   b) Describe Lagrange's method of undetermined multipliers. **(7+7)**

6. a) Define lower and upper Riemann integral of a function f(x). Also state the condition under which function is said to be Riemann integrable.

   b) Check whether following functions are Riemann integrable over (0, 1). If so find the integral.

      i) f(x) = 2x

      ii) f(x) = 2, if x is rational

      = 1, if x is irrational. **(7+7)**

7. a) Find liminf and limsup of the sequence $S_n = 1 + \frac{(-1)^n}{n}$. Hence discuss its convergence.

   b) Explain the term radius of convergence of a power series.

   c) State Lebnitz rule and its one application. **(8+3+3)**

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SLR-MM – 503

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – III)
Linear Algebra (New CBCS)

Day and Date : Friday, 20-11-2015 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q.No. **(1)** and Q. No. **(2)** are **compulsory.***
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) Which of the following sets of vectors are linearly dependent ?

$S_1 = \{(1, 2), (3, 4)\}$, $S_2 = \{(1, 2), (3, 4), (5, 6)\}$,

$S_3 = \{(1, 2, 3), (3, 4, 5)\}$, $S_4 = \{(1, 2, 3), (0, 0, 0)\}$

a) S_1 only b) S_2 and S_4 c) S_1 and S_3 d) S_4 only

2) Let A be a matrix A^{-1} exists if and only if A is a ___________ matrix.

a) Non-singular b) Square
c) Singular d) Real symmetric

3) The eigen values of a triangular matrix are ___________

a) Zero and one
b) The diagonal elements of the matrix
c) The off-diagonal elements of the matrix
d) None of these

4) If G is a g-inverse of A, then ___________

a) rank (G) $\leq$ rank (A) b) rank (G) $\geq$ rank (A)
c) rank (G) = rank (A) d) rank (G) $\leq$ rank (AG)

P.T.O.

5) The quadratic form $x_1^2 + x_2^2$ is __________

a) Positive definite
b) Negative definite
c) Positive semi-definite
d) Negative semi-definite **(1×5)**

B) Fill in the blanks:

1) The dimension of the vector space $V_3 = \{(x, x, y) : x, y \in (-\infty, \infty)\}$ is __________

2) A set of n + 2 vectors in n-dimensional Euclidean space is always linearly __________

3) Let A be an m×n matrix then the system of linear equations Ax = 0 has non-trivial solution if and only if __________

4) The eigen vectors of a symmetric matrix corresponding to different eigen values are __________

5) The matrix associated with the quadratic form $2x_1^2 + 3x_1x_2$ is ________ **(1×5)**

C) State **true** or **false** :

1) Let A and B be the two matrices. Then rank (A + B) $\leq$ min {rank (A), rank (B)}.

2) G inverse of a nonsingular matrix is unique.

3) All the eigen values of a non-singular matrix are non-zero.

4) Let p and q be the numbers of positive and negative d_i' s in the quadratic form $Q = \sum_{i=1}^{n} d_i x_i^2$, then Q is positive definite if and only if p = n. **(1×4)**

2. a) i) If X, Y, and Z are linearly independent vectors, examine whether U = X + Y, V = Y + Z, and W = X + Z are linearly independent.

ii) Prove or disprove that if λ is an eigen value of matrix A with corresponding eigen vector x then λ^m is an eigen value of A^m with corresponding eigen vector x for m = 2, 3, ... **(3+3)**

b) Write short notes on the following : **(4+4)**

i) Singular value decomposition

ii) Elementary row and column transformations of matrices.

3. a) Obtain orthonormal basis from the vectors a = (2, 0, 3), b = (1, 1, 0) and c = (0, 2,1) using Gram-Schmidt process of orthogonalization.

b) Show that any set of n linearly independent vectors in n-dimensional Euclidean space forms a basis for n-dimensional Euclidean space. **(7+7)**

4. a) Let A and B be m×n and n×p matrices, respectively. Show that rank (AB) $\leq$ min {rank(A), rank(B)}.

b) State and prove Caley-Hamilton theorem. **(7+7)**

5. a) Let $\lambda_1, \lambda_2, ..., \lambda_n$, be the characteristic roots of an n×n matrix A. Show that

$|A| \prod_{i=1}^{n} \lambda_i$ and trace (A) = $\sum_{i=1}^{n} \lambda_i$.

b) Show that if a real symmetric matrix A has eigen values 0 and 1 only then A is idempotent. **(7+7)**

6. a) Prove that matrix G is a g-inverse of matrix A if and only if AGA = A.

b) Consider a system of linear equations Ax = 0, where A is an m×n matrix of rank r(< n). Show that the number of linearly independent solutions to the system is n – r. **(7+7)**

7. a) Prove that the definiteness of a quadratic form is invariant under nonsingular linear transformation.

b) Reduce the following quadratic form to a form containing only square terms

$x_1^2 + x_3^2 + 4x_1x_3 + 8x_2x_3.$ **(7+7)**

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – IV)
Distribution Theory (New CBCS)

Day and Date : Monday, 23-11-2015 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) Suppose X is non-negative random variable and $Y = \frac{1}{X}$ then correlation between X and Y is

a) zero b) one c) positive d) negative

2) If X > 0 then

a) E [log X] = log [E (X)] b) E[log X] $\geq$ log [E(X)]

c) E[log X] $\leq$ log [E (X)] d) None of these

3) The m.g.f. of normal variable X is $M_x(t) = e^{2t+32t^2}$ then E (X^2) =

a) 15 b) 20 c) 32 d) 68

4) Suppose X has U (0, 1) distribution. The distribution of Y = – 2 log X is

a) uniform b) exponential c) chi-square d) normal

5) If X is geometric random variable on support {1, 2,...} then E (X) =

a) $\frac{1}{p}$ b) $\frac{1}{q}$ c) $\frac{q}{p}$ d) $\frac{p}{q}$

P.T.O.

B) Fill in the blanks : **5**

1) Let random vector $X = (X_1, X_2, ..., X_k)$ has $M (n, p_1, p_2, ..., p_k)$ distribution then distribution of X_i is __________
2) Dirichlet distribution is multivariate generalization of ________ distribution.
3) If X is continuous random variable with distribution function F(x) then Y = F(x) has ___________ distribution.
4) If X and Y are independent standard normal variates then $Var (X^2 + Y^2) =$ ______
5) If a random vector (X, Y) have a bivariate normal distribution then conditional distributions are

C) State whether the following statements are **true** or **false** : **4**

1) Moment generating function of a random variable is not necessarily unique.
2) If F(x) is distribution function then $[F(x)]^2$ is also a distribution function.
3) $\{N(\theta, 1), \theta \in R\}$ is a scale family.
4) Binomial distribution is a power series distribution.

2. a) Answer the following : **6**

i) Define Non-central F distribution.

ii) If $F_X (x)$ is distribution function of a continuous random variable X then find the distribution of $F_x (x)$.

b) Write short notes on the following : **8**

i) Convolution of distribution functions.

ii) Truncated binomial distribution.

3. a) Define : (i) Location family (ii) Scale family. Give one example of each.

b) Let F be a distribution of random variable X. Define $G (x) = \{F(x)\}^n$, n is positive integer. Examine G (x) to be distribution function. **(7+7)**

4. a) State and prove Minkowski's inequality.

b) Let X has N(0, 1) distribution. Find the distribution of $Y = X^2$. **(7+7)**

5. a) Define probability generating function (pgf) of a random variable. Explain how it is used to obtain moments of a distribution.

   b) Let X has Poisson distribution with parameter λ. Obtain pgf of X and hence its mean variance. **(7+7)**

6. a) Assume (X, Y) has trinomial distribution. Find the conditional expection of Y given X = x.

   b) Let X and Y are independent standard exponential random variates. Obtain the distribution of $\frac{X}{X+Y}$. **(7+7)**

7. a) Define Marshall-Olkin bivariate exponential distribution. State and prove forgetfulness property of the same.

   b) Let $X_1, X_2, \ldots, X_n$ are random observations from U(0, 1). Find the pdf of sample range. **(7+7)**

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SLR-MM – 505

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – V)
Estimation Theory (New CBCS)

Day and Date : Thursday, 26-11-2015 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Attempt* ***five*** *questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No. (**3**) to Q. No. (**7**).*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select the correct alternative :

1) If X_1, X_2 is a random sample from $P(\lambda)$ then the moment estimator of λ is

a) $X_1 + X_2$ b) $\frac{X_1 + X_2}{2}$ c) $2X_1 - X_2$ d) $\frac{(X_1 - X_2)^2}{2}$

2) Let $X \sim B(1, P)$ where $P \in \left[\frac{1}{4}, \frac{3}{4}\right]$. The MLE of θ on the basis of single observation X is

a) X b) $\frac{2X + 1}{4}$ c) $1 - X$ d) None of these

3) $X_1, X_2, \ldots X_n$ is random sample from a distribution with pdf

$$f(x) = \begin{cases} \theta\, x^{\theta - 1} & 0 < x < 1,\ \theta > 0 \\ 0 & \end{cases}$$

The sufficient statistic for θ is

a) $\prod_{l=1}^{n} X_i$ b) sample mean $\overline{X}$

c) max $(X_1 \ldots. X_n)$ d) min $(X_1, X_2, \ldots X_n)$

P.T.O.

4) Exponential distribution with pdf $f(x, \lambda) = \dfrac{e^{-\lambda} \lambda^{x}}{x!}$ $x = 0, 1, 2, ..., \lambda > 0$ is a member of

a) one parameter exponential family

b) power series family

c) both (a) and (b)

d) none of these

5) A prior distribution is the

a) distribution of sample X

b) distribution parameter θ

c) conditional distribution X given θ

d) none of these

B) Fill in the blanks :

1) A statistic whose distribution does not depend on the parameter is called as ________________

2) Method of scoring is used in ____________ estimation.

3) A minimum variance bound unbiased estimator __________ bound.

4) A minimal sufficient statistic is __________ complete.

5) Under quadratic error loss, Bayes estimator is __________ of posterior distribution.

C) State whether following statement is **true** or **false** :

1) MLE is always unbiased.

2) A sufficient statistics for θ base on r-s of size n from $U(0, \theta)$ is $\overline{X}$.

3) C-R lower bound is special case of Bhattacharya bound.

4) An unbiased estimator always exists. **(5+5+4)**

2. a) Explain the terms :

1) Sufficient and minimal sufficient statistic.

2) Minimum variance unbiased estimator.

b) Write short notes on the following :

1) Minimum chi-square method.

2) Basu's theorem. **(6+8)**

3. a) Explain and illustrate with examples the following term :
   i) complete statistic
   ii) information function.

   b) Obtain a minimal sufficient statistic for $U(\theta - 1, \theta + 1)$, $\theta \in \mathbb{R}$ based on a sample of size n. **(6+8)**

4. a) State and prove Rao-Blackwell theorem.

   b) State Lehmann-Scheffe theorem. Use it to derive MVUE of θ based on a random sample from exponential distribution with mean θ. **(7+7)**

5. a) State and prove Cramer-Rao inequality.

   b) Let $X_1, X_2, \ldots X_n$ be i.i.d. with pdf

$$f(x_1, \theta) = e^{-(x-\theta)} \quad x > \theta$$
$$= 0 \quad x < \theta$$

   Derive MLE for θ. **(7+7)**

6. a) Define prior distribution and posterior distribution. Illustrate each with one example.

   b) Describe the method moments for estimating unknown parameters. Let $X_1 \ldots X_n$ be iid from $U(0, \theta)$, obtain moment estimator for θ. **(7+7)**

7. a) State and prove invariance property of MLE.

   b) Let $X_1, X_2, \ldots X_n$ be a random sample from $N(\theta, \theta)$ distribution $0 < \theta < \infty$ obtain MLE of θ. **(7+7)**

SLR-MM – 508

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – III) (CGPA) (Old)
Linear Algebra

Day and Date : Friday, 20-11-2015 — Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt* ***five*** *questions.*
2) Q. No. ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No.* ***3*** *to Q. No.* ***7****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select the correct alternative :

i) Let V = {x, y, z, x, y, z$\in$ R} be a vector space then dimension of V is

A) 1 B) 2
C) 3 D) 0

ii) If A is an orthogonal matrix then

A) $A = A^T$ B) $A = A^{-1}$
C) $A = -A^T$ D) $A = A^2$

iii) Let A be an idempotent matrix. Then the value of $\max_X \frac{X^TAX}{X^TX}$ is ________

A) 1 B) 0
C) –1 D) ∞

iv) Let A and B be non-singular square matrices of the same order. Then which of the following is true ?

A) Rank (A) > Rank (B) B) Rank (A) < Rank (B)
C) Rank (A) $\neq$ Rank (B) D) Rank (A) = Rank (B)

v) Consider the following system of equations :

$x + y = 3,\ x - y = 1,\ 2x + y = 5.$

The above system has

A) Unique solution B) No solution
C) More than one solution D) None of these

B) Fill in the blanks :

i) The rank of a $K \times K$ orthogonal matrix is __________

ii) A superset of linearly dependent set of vectors is linearly __________

iii) If the trace and determinant of a 2×2 matrix are 5 and 6, then smallest characteristic root is __________

iv) Any square matrix can be written as sum of symmetric and __________

v) If $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then M^{-1} = __________

C) State whether following statements are **true** or **false** :

I) A matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ is positive definite matrix.

II) Every matrix has a unique g-inverse.

III) The symmetric matrix A of the quadratic form $(x_1 + x_2)^2$ is $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

IV) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $a \neq 0$, then $G = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & 0 \end{bmatrix}$ is always a g-inverse of A.

(5+5+4)

2. a) Answer the following :

a) Define vector space with illustration.

b) Define inverse of a matrix. Prove that $(AB)^{-1} = B^{-1}A^{-1}$.

b) Write short notes on the following :

a) Elementry operations on a matrix.

b) Moore-Penrose (MP) inverse. **(6+8)**

3. a) Define (I) Dimension of a vector space (II) Basis of a vector space. Prove that any two bases of vector space contain same number of vectors.

b) Define linearly independent and dependent set of vectors. Examine whether the following set of vectors is linearly independent.
$a_1 = (1, 1, 2)$ $a_2 = (2, 2, 3)$ $a_3 = (1, 2, 3)$. **(7+7)**

4. a) Define and illustrate one example each, the folloiwng terms :

   I) Rank of a matrix

   II) Kroneckar product of two matrices

   III) Skew symmetric matrix.

   b) Let N be a non-singular matrix of order n partitioned as $N = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}$, where N_{22} is a non-singular matrix of order m (m < n). Obtain inverse of N. **(6+8)**

5. a) Define g-inverse of matrix. Show that $\overline{A}$ is a g-inverse of A iff $A\overline{A}A = A$.

   b) Show that a system of linear equations $A\underline{X} = b$ is consistent iff $\rho(A|b) = \rho(A)$. **(6+8)**

6. a) Define characteristic roots and vectors of a matrix show that the characteristic vector corresponding to the distinct characteristic roots of real symmetric matrix are orthogonal.

   b) Prove Cayley-Hamilton theorem. Indicate how can be used to find inverse of a given matrix. **(7+7)**

7. a) Define a quadratic form. Give an example. Show that quadratic form is invariant under non-singular transformation.

   b) Reduce the following quadratic form $x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 + 2x_2x_3 - 2x_1x_3$ to canonical form and determine whether it is definite or indefinite. **(7+7)**

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Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – IV)
(Old CGPA)
Distribution Theory

Day and Date : Monday, 23-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) Let X be a $N(\mu, \sigma^2)$ variable. Then distribution of e^x is

a) $N(0, \sigma^2)$ b) Lognormal
c) Half normal d) Standard normal

2) Let X be distributed as B(n, p). The distribution of Y = n – X is

a) Not Binomial b) B(n, p)
c) B(n, n – p) d) B(n, 1 – p)

3) Let X be distributed as Exp (Mean θ). Then distribution of $Y = \frac{X}{\theta}$ is

a) Exp (Mean θ) b) Exp (Mean 1)
c) U(0, 1) d) U(0, θ)

4) Let X be a non-negative random variable (r.v.) with distribution function F. If E(X) exists, then E(X) =

a) $\int_0^\infty F(x)\,dx$ b) $\int_0^\infty [F(x)-1]\,dx$

c) $\int_0^\infty [1-F(x)]\,dx$ d) $\int_0^\infty [1+F(x)]\,dx$

5) The probability generating function (p.g.f) of geometric distribution with parameter p is $P_X(S) =$

a) $\dfrac{p}{(1-qS)}$ b) $\dfrac{q}{(1-pS)}$

c) $\dfrac{p}{\left(1-\dfrac{q}{S}\right)}$ d) $\dfrac{q}{\left(1-\dfrac{p}{S}\right)}$

B) Fill in the blanks : **5**

1) Dirichlet distribution is multivariate generalization of _________ distribution.

2) The coefficient of S^k in the expansion of $P_X(S)$ gives _________

3) Holder's inequality is given as _________

4) If X is symmetric about α then 1 – X is symmetric about _________

5) Let $X_1, X_2, \ldots, X_n$ are i.i.d. $N(\mu, \sigma^2)$ random variables and $\overline{X}$ is sample mean. Then distribution of $\overline{X}$ is _________

C) State whether the following statements are **True** or **False** : **4**

1) M.g.f. of Cauchy random variable does not exist.

2) Order statistics are independent.

3) If X>0 then $E\left[\sqrt{X}\right] \le \sqrt{E[X]}$.

4) Liapounov's inequality is special case of Markov inequality.

2. a) Answer the following : **6**

i) Define location family. Give one example illustrating it.

ii) Let X has N(0, 1) distribution. Find the distribution of X^2.

b) Write short notes on the following : **8**

i) Bivariate Poisson distribution.

ii) Non-central t distribution.

3. a) State and prove the relation between distribution function of continuous random variable and uniform random variable.

   b) Decompose the following distribution function into discrete and continuous components.

$$F(x) = \begin{cases} 0, & x < 0 \\ \dfrac{1}{2}, & x = 0 \\ \dfrac{1}{2} + \dfrac{x}{2}, & 0 < x < 1 \\ 1, & x \geq 1 \end{cases}$$

(7+7)

4. a) State and prove basic inequality.

   b) State and prove the use of moment generating function for obtaining moments of random variable. **(7+7)**

5. a) Define order statistics. Based on a random sample from continuous distribution with p.d.f. f(x) and d.f. F(x), derive the p.d.f. of (i) smallest order statistic (ii) largest order statistic.

   b) Define convolution of distribution functions and give one example. **(8+6)**

6. a) Let X be a random variable with probability mass function (p.m.f.)

   $P(X = x) = \dfrac{e^{-\lambda} \lambda^{x}}{x!}$, x = 0, 1, 2, . . . and $\lambda > 0$.

   Suppose that the value of x = 0 cannot be observed. Find the p.m.f. of truncated random variable and its mean.

   b) If X and Y are jointly distributed with probability density function (p.d.f.)

   f(x, y) = x + y, $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Find $P\left[X > \sqrt{Y}\right]$. **(7+7)**

7. a) Define multinomial distribution. Obtain its moment generating function. Hence or otherwise find the variance-covariance matrix.

   b) Let X and Y are independent N(0, 1) variates. Show that $E\left[\text{Max.}(X, Y)\right] = \dfrac{1}{\sqrt{\pi}}$.

(7+7)

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – V) (CGPA) (Old)
Estimation Theory

Day and Date : Thursday, 26-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt* ***five*** *questions.*

2) Q.No. ***(1)*** *and Q.No.* ***(2)*** *are* ***compulsory****.*

3) Attempt ***any three*** *questions from Q.No.* ***(3)*** *to Q.No.* ***(7).***

4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select the correct alternative :

1) Let T_n be an unbiased estimator of θ. Then

a) T_n^2 is unbiased estimator of θ^2

b) $\sqrt{T_n}$ is unbiased estimator of $\sqrt{\theta}$

c) e^{T_n} is unbiased estimator of $e\theta$

d) $3T_n + 4$ is unbiased estimator of $3\theta + 4$.

2) Suppose $X_1, X_2 ..., X_n$ is a random sample from a distribution with finite mean μ. The MME for estimating μ is

a) $\sum_{i=1}^{n} X_i$ b) $\frac{1}{n}\sum_{i=1}^{n} X_i$

c) $\sum_{i=1}^{n} (X_i - \overline{X})$ d) None of above

3) MLE is always

   a) unique

   b) unbiased

   c) A function of sufficient statistic

   d) complete statistic

4) _____ is not a one-parameter exponential family.

   a) B (n, θ) b) N (θ, 1) c) C (1, θ) d) P (θ)

5) Let $X_1, X_2 ..., X_n$ be a random sample from $N(\mu, \sigma^2)$. Then a sufficient statistic for σ^2 when μ is known is

   a) $\frac{1}{n-1}\sum\limits_{i=1}^{n} (X_i - \overline{X})^2$ b) $\sum\limits_{i=1}^{n} (X_i - \mu)^2$

   c) $\sum\limits_{i=1}^{n} X_i^2$ d) $\left(\sum\limits_{i=1}^{n} X_i, \sum\limits_{i=1}^{n} X_i^2\right)$ **(1×5)**

B) Fill in the blanks :

1) Suppose T is an unbiased estimator of θ. Then g (T) is unbiased for g(θ) if g is a ______ function.

2) If X_1, X_2 are iid $N(\mu, 1)$ random variables, then $X_1 - X_2$ is _____ statistic for μ.

3) For power series family of distribution _____ is sufficient statistic for θ.

4) The statistic used in minimum chi-square method to estimate a parameter is _______

5) The Bayes estimator of parameter θ under absolute error loss is ____ **(1×5)**

C) State **True** or **false** :

1) MLE is always unique.

2) Bayes estimators are functions of sufficient statistics.

3) Every function of a minimal sufficient statistic is minimal sufficient.

4) Crammer-Rao lower bound is a particular case of Bhattacharya bound. **(1×4)**

2. a) i) Describe the method of minimum chi-square.

ii) Show that every one-to-one function of a sufficient statistic is also sufficient. **(3+3)**

b) Write short notes on the following :

i) Posterior distributions.

ii) Completeness and bounded completeness. **(4+4)**

3. a) Define UMVUE. Obtain UMVUE of P (X = 1) based on a random sample of size n, where X has $P(\lambda)$ distribution.

b) State and prove a necessary and sufficient condition for an estimator of a parametric function $\psi(\theta)$ to be UMVUE. **(7+7)**

4. a) Define one-parameter exponential family of distributions. Obtain a minimal sufficient statistic for this family.

b) Suppose an observation is taken on random variable X which yielded a value 2. The density of X is $f(x/\theta) = \frac{1}{\theta}, 0 < x < \theta$. Suppose prior distribution of θ has density $\pi(\theta) = \frac{3}{\theta^4}, \theta > 1$. For the squared error loss function, show that Bayes estimate of θ is $\frac{8}{3}$. **(7+7)**

5. a) Define MLE. Show that an MLE, if exists, is a function of sufficient statistic.

   b) Let $X_1, X_2 ..., X_n$ be a random sample from exponential distribution with location parameter μ and scale parameter σ. Obtain moment estimates of (μ, σ). **(7+7)**

6. a) State and prove Basu's theorem. Give its one application.

   b) State Cramer-Rao inequality. Give two examples of estimators such that the mean square error of one attains Cramer-Rao lower bound while that of the other does not. **(7+7)**

7. a) Define completeness. Prove or disprove that $\{U(0, \theta), \theta \in (0, \infty)\}$ is a complete family.

   b) State and prove Chapman-Robbins-Kiefer inequality. **(7+7)**

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VII)
Linear Models (New) (CGPA)

Day and Date : Thursday, 19-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*

*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*

*3) Attempt **any three** from Q. No. **(3)** to Q. no. **(7)**.*

*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) In general linear model, $y = X\beta + \varepsilon$

a) $\text{rank}[X'X, X'y] = \text{rank}\ [X'X]$ b) $\text{rank}[X'X, X'y] \leq = \text{rank}\ [X'X]$

c) $\text{rank}[X'X, X'y] \geq = \text{rank}\ [X'X]$ d) $\text{rank}[X'X, X'y] < \text{rank}\ [X'X]$

2) In one-way ANOVA model $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$; $i = 1, 2, \ldots..k$; $j = 1, 2, \ldots n_i$, the dimension of estimation space is

a) $k - 1$ b) n_i c) $n_i - 1$ d) k

3) In two-way ANOVA model $y_{ij} = \mu + \alpha_i + \beta_i + \varepsilon_{ij}$; $i = 1, 2, \ldots p$; $j = 1, 2, \ldots q$ the test statistic for testing the equality of β_j's has F distribution with .....d.f.

a) $(p - 1), (p - 1)(q - 1)$ b) $(q - 1), (p - 1)(q - 1)$

c) $(p - 1), pq - p - q + 2$ d) $(p - 1), pq - p - q - 2$

4) A balanced design is ____________ connected.

a) sometimes b) always c) never d) generally

5) For a BIBD with usual notation, $\lambda(v - 1) =$

a) $k(r - 1)$ b) $k(r + 1)$ c) $r(k + 1)$ d) $r(k - 1)$

(1×5)

B) Fill in the blanks :

1) In general linear model $y = X\beta + \in$, the quantity $XS^{-}X'$ is __________ under the choice of g-inverse of $S = X'X$.

2) In general linear model, $y = X\beta + \varepsilon, V(\lambda'\beta) =$ __________

3) A connected block design can not be ______________

4) A block design is ___________ if and only if $CR^{-\delta}N = 0$.

5) The degrees of freedom of error SS in two-way without interaction ANOCOVA model with p rows, q columns, 1 observation per cell, and m covariate is __________ **(1×5)**

C) State **true** or **false**.

1) The degree of freedom of error SS in two-way ANOVA with interaction model with p rows and q columns and with one observation per cell is one.

2) In general linear model, any linear function of the LHS of normal equations is the BLUE of its expected value.

3) BIBD is not orthogonal.

4) A connected design is always balanced. **(1×4)**

2. a) i) Show that any solution of normal equations minimizes the residual sum of squares.

ii) Examine whether the following block design is connected.

$$B_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \text{ and } B_3 = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}.$$ **(3+3)**

b) Write short notes on the following :

i) Tuckey's test of non-additivity

ii) Dual of a BIBD. **(4+4)**

3\. a) Prove that in general model $y = X\beta + \in$, the BLUE of every estimable linear parametric function is a linear function of the LHS of normal equations, and conversely, any linear function of the LHS of normal equations is the BLUE of its expected value.

b) Prove that in general linear model $y = X\beta + \in$, a necessary and sufficient condition for the estimability of a linear parametric function $\lambda'\beta$ is that $\lambda' = \lambda' H$, where $H = S^{-}S, S = X'X$. **(7+7)**

4\. a) Derive the test for testing the hypothesis of the equality of treatment effects in one-way ANOVA model.

b) Describe two-way ANOVA without interaction model with one observation per cell and obtain the least square estimates of its parameters. **(7+7)**

5\. a) Describe Tuckey's and Scheff's procedures of multiple comparisons.

b) Describe ANOCOVA model is general and obtain the least square estimates of its parameters. **(7+7)**

6\. a) Derive a test for testing a general linear hypothesis in a general linear model.

b) prove that RBD is connected, orthogonal and balanced. **(7+7)**

7\. a) State and prove a necessary and sufficient condition for orthogonality of a connected block design.

b) Prove that in a BIBD, the number of blocks is greater than or equal to the number of treatments. **(7+7)**

Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
STATISTICS (Paper – VIII)
Stochastic Processes (New) CGPA

Day and Date : Saturday, 21-11-2015 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** from Q. **3** to **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the most correct answer :

1) Let X_n denotes maximum temperature on n^{th} day then $\{X_n, n \geq 0\}$ is a stochastic process with

a) Discrete time, discrete state space
b) Discrete time, continuous state space
c) Continuous time, discrete state space
d) Continuous time continuous state space

2) Let $\{x_n, n = 0, 1, ...\}$ be an MC with state space {0, 1} and tpm $p = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$.

Which of the following is true ?

a) State 1 is aperiodic b) State 1 is absorbing
c) Both a) and b) d) Neither a) nor b)

3) State i is recurrent iff

a) $\sum_{n=0}^{\infty} P_{ii}^{(n)} < \infty$ b) $\sum_{n=0}^{\infty} P_{ii}^{(n)} = 1$ c) $\sum_{n=0}^{\infty} P_{ii}^{(n)} = \infty$ d) $\sum_{n=0}^{\infty} P_{ii}^{(n)} = 0$

4) Let $\{X_n, n = 0, 1, 2, ...\}$ be a branching process. If $E(X_1) = m$ then $E(X_n)$ is

a) n^m b) m^n c) mn d) mn^m

5) The renewal function M(t) is

a) $M(t) = \frac{d}{dt} E(N(t))$ b) $M(t) = E(N^2(t))$

c) $M(t) = E(N(t))$ d) None of these

B) Fill in the blanks :

1) An aperiodic non-null persistent state is called as __________

2) State i is said to be accessible from j if __________ for some n.

3) If N_1 (t) and N_2(t) are two independent Poisson processes with parameters λ_1 and λ_2 respectively then $N_1(t) + N_2(t)$ is __________

4) In Yule-Furry birth process the birth rate λ_n is __________

5) In a Galton-Watson branching process, if the mean offspring m is less than one, then the probability of ultimate extinction is __________

C) State whether following statements are **true** or **false** :

1) Stationary distribution need not be unique.

2) Number of accidents during 0 to t(x) can be well modeled by birth and death process.

3) Delayed renewal process is a particular case of renewal process.

4) Traffic intensity is defined as $\frac{\text{arrival rate}}{\text{service rate}}$. **(5+5+4)**

2. A) Answer the following :

i) What is first passage time distribution ?

ii) What is steady state distribution ? **(3+3)**

B) Write short notes on the following :

i) Finite dimensional distributions of stochastic processes.

ii) Renewal process. **(4+4)**

3. A) Let $\{X_n, n = 0, 1, ..\}$ be a M.C. with state space {0, 1, 2}, t.p.m.

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.3 & 0 & 0.7 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$ and initial distribution is $\left(\frac{1}{3}, 0, \frac{2}{3}\right)$

Find :

i) Marginal distribution of X_2

ii) $P(X_1 = 1, X_2 = 2)$

iii) Mean recurrence time of state 1.

B) Define :

1) Homogeneous Markov chain

2) Irreducible Markov chain

3) Class property. **(9+5)**

4. A) Explain random walk model in detail.

B) Define stationary distribution. Obtain the same if $\{X_n, n = 0, 1,...\}$ be a MC with tpm

$$\begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix}.$$ **(7+7)**

5. A) Obtain difference differential equation of pure birth process.

B) Define Poisson process. If $\{N(t)\}$ is a Poisson process then find the auto correlation coefficient between $N(t)$ and $N(t + s)$, $s, t > 0$. **(7+7)**

6. A) For a renewal process {N(t), t $\geq$ 0}, show that

$$\frac{N(t)}{t} \to \frac{1}{\mu} \text{ as } t \to \infty \text{ with probability 1 if } \mu = E(X_n) < \infty.$$

B) Show that probability of ultimate extinction is the smallest root of the equation $\phi(S) = S$ where $\phi(S)$ is the p.g.f. of offspring distribution. **(7+7)**

7. A) Let {X_n, n = 0, 1, ...) be a M.C. with state space {0, 1, 2}, tpm

$$p = \begin{bmatrix} 0.4 & 0 & 0.6 \\ 0.6 & 0 & 0.4 \\ 0.2 & 0.6 & 0.2 \end{bmatrix}$$

and initial distribution is $\left(\frac{1}{3}, \frac{2}{3}, 0\right)$, write an algorithm for the simulation of given MC.

B) Explain M/M/1 queuing model with suitable example. **(7+7)**

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SLR-MM – 514

Seat No.

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – IX)
Theory of Testing of Hypotheses (New) (CGPA)

Day and Date : Tuesday, 24-11-2015 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the right indicate **full** marks.*

1. A) 1) Let $X \sim U(0, \theta)$, $H_0 : \theta = 10$, $H_1 : \theta = 15$. To test H_0 against H_1, consider a non-randomised test with acceptance region (0, 5) then α and β the first and second kind of errors respectively are given by

A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ B) $\left(\frac{1}{2}, \frac{2}{3}\right)$

C) $\left(\frac{2}{3}, \frac{2}{3}\right)$ D) None of these

2) ϕ_1 and ϕ_2 are two test functions then which one of the following is not a test function

A) $\lambda\phi_1 + (1-\lambda)\phi_2$, $0 \le \lambda \le 1$ B) ϕ_1^2

C) $2\phi_1$ D) $\phi_1 . \phi_2$

3) A test function $\phi(x) = 0.5$ for all x has power

A) 1 B) 0

C) 0.5 D) None of these

4) If $X_1, X_2.... X_n$ are iid exponential rv's with unknown θ then family has an MLR property in

A) $\sum\limits_{1=1}^{n} Xi$ B) $\sum\limits_{1=1}^{n} Xi^2$

C) $\max(X_1, X_2, ... X_n)$ D) $\min(X_1, X_2, ... X_n)$

P.T.O.

5) Identify the pivotal quantity from the following when X_1, X_2 is a random sample from $N(\theta, 1)$, $\theta \in R$.

A) $X_1 + X_2$ B) $X_1 - X_2$

C) $X_1 + X_2 - \theta$ D) $X_1 + X_2 - 2\theta$

B) Fill in the blanks :

1) Neyman Pearson lemma is used to obtain _______ test.

2) Type II error is accepting H_0 when H_1 is ______

3) The degrees of freedom associated with 5×4 contingency table is

4) The class of UMPU test is a sub class of ______

5) A non-randomized test function takes values either ______ or ______.

C) State whether the following statements are true (T) and false (F).

1) The most powerful test is biased.

2) The family of Cauchy distribution satisfy MLR property.

3) Let $X_1, X_2 \ldots X_n$ are iid exponential with mean θ, the quantity $\theta \sum_{1=1}^{n} Xi$ is the pivotal quantity.

4) Likelihood test is always unbiased. **(5+5+4)**

2. a) Explain :

I) Simple and composite hypothesis

II) Power and size of the test

Give one example each.

b) Write short notes on the following :

I) Run test

II) Likelihood ratio test. **(6+8)**

3. a) Define Most Powerful (MP) test. Explain the method of obtaining test of size α for testing simple hypothesis against simple alternative.

b) Obtain the M.P. test of size α for testing the hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$ based on a random sample of size n from Poisson distribution $P(\theta)$. **(7+7)**

4. a) Describe the Mann-Whitney test, stating clearly the null hypothesis, alternative hypothesis. Derive the null distribution of Mann-Whitney U-statistic.

   b) Define UMP test. Let $X_1, X_2, ... X_n$ from $U(\theta, \theta)$, discuss the existence UMP test for testing $H_0 : \theta = \theta_0$ vs $H_1 : \theta \neq \theta_0$. **(7+7)**

5. a) Explain :

   I) Monotone likelihood ratio property

   II) Unbiased test and UMPU test.

   b) Let $X_1, X_2, ... X_n$ denote a random sample from $f(x, \theta) = \theta x^{\theta^{-1}}$, $0 < x < 1$, $\theta > 0$. Derive UMPU test for testing $H_0 : \theta = 1$ against $\theta \neq 1$. **(7+7)**

6. a) Explain :

   I) UMA and UMAU confidence interval.

   II) Shortest length confidence interval.

   b) Let $X_1, X_2, ... X_n$ be a random sample from $N(\mu, \sigma^2)$, when μ is unknown. Obtain shortest length confidence interval $(1 - \alpha)$ level for σ^2. **(7+7)**

7. a) Let $X_1, X_2 .... X_n$ be a random sample from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown. Obtain LRT for testing $H_0 : \mu = \mu_0$ is $H_1 : \mu \neq \mu_0$.

   b) Discuss :

   1) Generalised Neyman Pearson lemma.

   2) Sign test. **(7+7)**

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Seat No.	

M.Sc. (Part – I) (Sem. – II) Examination, 2015
(Old – CGPA)
STATISTICS (Paper – VII)
Linear Models and Design of Experiments

Day and Date : Thursday, 19-11-2015 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative : **5**

1) Let $E(Y_1) = \theta_1 + \theta_2$, $E(Y_2) = \theta_1 - \theta_2$, $E(Y_3) = \theta_1$, $V(Y_i) = \sigma^2$; $i = 1, 2, 3$. Let $\hat{\theta}_i$ be the least square estimator of θ_i. Which of the following statement is correct ?

a) $\hat{\theta}_1 = \dfrac{Y_1 + Y_2 + Y_3}{2}$ b) $\hat{\theta}_2 = \dfrac{Y_1 + Y_2 + Y_3}{2}$

c) $V(\hat{\theta}_1) = \dfrac{3\sigma^2}{4}$ d) $V(\hat{\theta}_2) = \dfrac{\sigma^2}{2}$

2) An experimenter wishes to compare 8 treatments in blocks of size 4, using BIBD with 14 blocks, then any pair of treatments appear together in _________ blocks.

a) 4 b) 3 c) 2 d) 1

3) In a connected block design with v treatments and b blocks, rank of D matrix is

a) v – 1 b) b – 1 c) v + 1 d) b + 1

P.T.O.

4) In the linear model $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$, i = 1, 2, 3, j = 1, 2, 3, 4, the number of linearly independent estimable parametric functions is

a) 12 b) 3 c) 4 d) 7

5) From an RBD with 4 treatments and 5 blocks one block is removed, then the resulting design is

a) CRD b) BIBD c) LSD d) RBD

B) Fill in the blanks : **5**

1) In a general linear model, the covariance between any linear function belonging to the error space and any BLUE is ___________

2) The BLUE of a treatment contrast $\sum_i c_i \alpha_i$ in one-way ANOVA model is ________

3) The degrees of freedom of error SS in two-way without interaction ANOCOVA model with p rows, q columns, 1 observation per cell and 1 covariate is __________

4) In a general block design, covariance between adjusted treatment totals and block totals is _________

5) In a general linear model, $\underline{y} = x\underline{\beta} + \underline{\varepsilon}$, a linear parametric function $\underline{\lambda}' \underline{\beta}$ is estimable if and only if $\underline{\lambda}'$ = _________

C) State **true** or **false** : **4**

1) In a general linear model, the normal equations are always consistent.

2) Individual parameters are not estimable in one-way and two-way ANOVA models.

3) A connected design is necessarily balanced.

4) BIBD is not orthogonal.

2. a) Define BIBD and show that it is connected, non-orthogonal and balanced.

b) Show that for BIBD with parameters (v, b, r, k, λ), $b \geq v$. **(7+7)**

3. Consider $E(Y_1) = \theta_1 + \theta_2 + \theta_3$, $E(Y_2) = E(Y_4) = \theta_1 - \theta_4$, $E(Y_3) = \theta_1 + \theta_2$ and $cov(\underline{Y}) = \sigma^2 I_n$.

a) Check whether the above model is full-rank model.

b) Obtain rank of the estimation space and rank of the error space.

c) Obtain one solution of normal equations and hence obtain BLUE of $\theta_1 + \theta_2$ if it is an estimable parametric function. **(4+4+6)**

4. a) Explain Tukey's method of comparison of k different individual means.

   b) Prove or disprove BLUE is unique.

   c) For the linear model, $\underline{y} = x\underline{\beta} + \underline{\varepsilon}$, explain conditional and unconditional sum of squares of error. **(4+4+6)**

5. a) Given below is the incidence matrix N_A of design A. Check whether the design is connected.

$$N_A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

   b) In a general block design, show that row sums and column sums of C matrix are all zero.

   c) Obtain BLUE of $\sum_i l_i \alpha_i$, $\sum_i l_i = 0$ in the one-way ANOCOVA model with single covariate. **(3+3+8)**

6. A) Write the linear model for one-way classification with one concomitant variable. Obtain the least square estimates of its parameters. **14**

   B) Derive the test for testing the hypothesis of the equality of row effects in two-way ANOVA without interaction model with one observation per cell.

7. a) Explain the following terms :

      i) Estimation space

      ii) BIBD and its properties.

   b) In a general linear model $\underline{y} = x\underline{\beta} + \varepsilon$, develop a test for testing $H_0 : \wedge \underline{\beta} = \underline{0}$. **(8+6)**

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Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – IX) (CGPA) (Old)
Theory of Testing of Hypotheses

Day and Date : Tuesday, 24-11-2015 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Attempt **five** questions.*
*2) Q.No. (**1**) and Q.No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q.No. (**3**) to Q.No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative : **5**

1) Which of the following is simple hypothesis for $N(\theta, \sigma^2)$?

a) $H_0 : \theta = 10$ b) $H_0 : \theta = 0, \sigma > 1$

c) $H_0 : \theta = 5, \sigma = 2$ d) $H_0 : \theta \neq 3, \sigma = 1$

2) If α and β are probabilities of type I and type II errors respectively. Which of the following inequality is satisfied by MP test ?

a) $\alpha < \beta$ b) $\alpha > \beta$ c) $\alpha + \beta > 1$ d) $\alpha + \beta \leq 1$

3) Degrees of freedom for a χ^2 in case of contingency table of order (4×3) are

a) 3 b) 6 c) 9 d) 12

4) The p.d.f. $f(x) = \frac{1}{2} e^{-|x-\theta|}, -\infty < x < \infty$ has MLR in

a) x b) $-x$ c) $|x|$ d) x^2

5) For LRT, asymptotic distribution of $-2\log\lambda$ is

a) normal b) t c) F d) χ^2

P.T.O.

B) Fill in the blanks. **5**

1) A good confidence set should have ________ length.
2) Based on single observation x from logistic distribution has MLR in ________
3) Probability of rejecting H_0 when it is false is called _________ of test.
4) Let $X \sim U(0, \theta)$. Then $H : \theta \leq 5$ is __________ hypothesis.
5) Acceptance region of _________ test leads to UMA confidence set.

C) State whether the following statements are **true** or **false**. **4**

1) Cauchy (1, θ) posses MLR property.
2) LRT is UMPU test.
3) A class of α-similar tests is a subclass of all unbiased size α tests.
4) If ϕ is randomized test then $(1 - \phi)$ is also randomized test.

2. a) Answer the following : **6**

1) Define :
   i) Size of test
   ii) Power of test
2) Explain likelihood ratio test procedure.

b) Write short notes on the following : **8**

i) Sign test
ii) Unbiased test.

3. a) State Neyman-Pearson Lemma and prove sufficient condition for a test to be most powerful.

b) Let X be a discrete random variable having two possible p.m.f.s given by

X	:	0	1	2	3	4
$P_0(x)$	:	0.2	0.3	0.1	0.1	0.3
$P_1(x)$	:	0.1	0.2	0.2	0.2	0.3

Obtain MP test of size $\alpha = 0.05$ for testing $H_0 : X \sim P_0(x)$ against $H_1 : X \sim P_1(x)$ on the basis of random sample of size one. Also compute power of test. **(7+7)**

4. a) Define MLR property of a family of distributions. Explain the use of MLR in construction of UMP test with the help of suitable example.

   b) Let $X_1, X_2, ..., X_n$ be a random sample of size n from $N(\theta, 1)$. Obtain UMP level α test for $H_0 : \theta \leq \theta_0$ against $H_1 : \theta \leq \theta_0$. **(7+7)**

5. a) Define UMPU test. Prove that every UMP test is UMPU of same size.

   b) Let $X_1, X_2, ... X_n$ be iid $U(0, \theta)$. Consider the following test for the $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

$$\phi(x) = \begin{cases} 1, \text{ if } x_{(n)} > \theta_0 \text{ or } x_{(n)} < \theta_0 \, \alpha^{\frac{1}{n}} \\ 0, \text{ otherwise} \end{cases}$$

   Examine whether ϕ is UMP. **(6+8)**

6. a) Define UMA confidence interval. Obtain one sided confidence interval for θ based on n independent observations from exponential distribution with mean θ.

   b) Derive LRT test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ based on random sample of size n from $N(\theta, 1)$ distribution. **(7+7)**

7. a) Describe the test for independence of attributes.

   b) Describe Wilcoxon's signed-rank test. **(7+7)**

SLR-MM – 521

Seat No.	

M.Sc. – II (Semester – III) (CGPA) Examination, 2015
STATISTICS (Paper No. – XI)
Asymptotic Inference (New)

Day and Date : Monday, 16-11-2015 Total Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** from Q. **3** to **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternatives of the following questions.

i) Let T_n be the sequence of consistent estimators of θ, then

a) $\lim_{n\to\infty} Var(T_n) = 0$ b) $\lim_{n\to\infty} E(T_n - \theta)^2 = 0$

c) both a and b d) neither a nor b

ii) Let $X_1, X_2 ... X_n$ be iid from B (1, θ), then asymptotic distribution of $\overline{X}(1-\overline{X})$ is

a) Normal with mean $\theta(1-\theta)$ and variance $\frac{\theta(1-\theta)}{n}(1-2\theta)^2$, if $\theta \neq \frac{1}{2}$

b) Normal with mean $\theta(1-\theta)$ and variance $\frac{\theta(1-\theta)}{n}(1-2\theta)^2$, for all θ

c) Normal with mean $\theta(1-\theta)$ and variance $\frac{\theta(1-\theta)}{n}$, for all θ

d) None of the above

iii) Let $X_1, X_2 ... X_n$ be iid from $N(\theta, \sigma^2)$, then

a) $\overline{X}$ is unique consistent estimator of θ

b) $\overline{X}$ and sample median are the only two consistent estimators for θ

c) No consistent estimator exist for θ if σ^2 is unknown

d) There are infinitely many consistent estimators for θ

P.T.O.

iv) Let $X_1, X_2 ... X_n$ be iid from $U(0, \theta)$ then

a) $X_{(n)}$ is CAN

b) $X_{(n)}$ is consistent and but not asymptotically normal

c) $X_{(n)}$ is consistent and BAN

d) None of the above

v) Let $X_1, X_2 ... X_n$ be iid $\exp(\beta, \sigma)$, then

a) $X_{(1)}$ is MLE of β

b) S^2 is MLE of σ

c) $X_{(1)}$ is CAN estimator β

d) MLE of σ does not exists

B) Fill in the blanks.

i) Let $X_1, X_2 ... X_n$ be iid from Cauchy (θ, 1) distribution, then consistent estimator of θ is ______________.

ii) Asymptotic distribution of sample distribution function is ________ provided underlying distribution is continuous.

iii) If distribution of X belong to one parameter exponential family of distributions then the moment estimator of θ based on sufficient statistic is __________ estimator.

iv) Wald test statistic for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ is _________.

v) Asymptotic variance of super efficient estimator is _______ than fisher lower bound for some $\theta = \theta_0$ and equal to fisher lower bound otherwise.

C) State whether following statements are **true** or **false**.

i) Let $X_1, X_2 ... X_n$ be iid from $P(\theta)$, then $\overline{X}$ is unique consistent estimator of θ.

ii) Let $X_1, X_2 ... X_n$ be iid from $f(x, \theta)$, and $T = T(X_1, X_2 ... X_n)$ be a CAN estimator for θ, if g (.) is continuous, differentiable function the g (T) is CAN estimator for $g(\theta)$ provided $\frac{dg(\theta)}{d\theta} \neq 0$.

iii) Cramer family of distributions is subset of exponential family of distribution.

iv) Log transformation is the variance stabilizing transformation for a $\exp(\theta)$ population. **(5+5+4)**

2. a) Write a note on method of moments estimation for CAN estimator.

   b) Let $X_1, X_2...X_n$ be iid $\exp(\theta, \sigma)$, then show that $\left(X_{(1)}, \sum_{i=1}^{n}(X_i - X_{(1)})\right)'$ is jointly consistent for $(\theta, \sigma)'$.

   c) Describe Rao's score test.

   d) Write a note on super efficient estimator. **(4+4+3+3)**

3. a) Define weak and strong consistency. Obtain consistent estimator for mean of double exponential distribution based on sample of size n.

   b) Define asymptotic relative efficiency. Let $X_1, X_2...X_n$ be iid from $U(0, \theta)$, obtain relative efficiency of $X_{(n)}$ to $2\overline{X}$. **(7+7)**

4. a) Define one parameter Cramer family of distributions and show that in one parameter Cramer family, with probability approaches to 1 as $n \to \infty$, the likelihood equation admits consistent solution.

   b) Let $X_1, X_2...X_n$ be iid $N(\theta, \theta)$, then obtain three consistent estimators for θ. **(10+4)**

5. a) Let $X_1, X_2...X_n$ be iid Cauchy $(\theta, 1)$, then find the asymptotic distribution MLE of θ and asymptotic variance.

   b) Let $X_1, X_2...X_n$ be iid $\exp(\theta)$, θ mean, using show that $\overline{X}$ is CAN estimator for θ and obtain a CAN estimator for $P(X > t)$ and its asymptotic variance. **(7+7)**

6. a) Let $X_1, X_2...X_n$ be iid $B(1, \theta)$. Construct $100\ (1 - \alpha)$ level VST confidence interval for θ.

   b) Let $X_1, X_2...X_n$ be iid $N(\theta, 1)$, then derive LRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. **(7+7)**

7. a) Derive Bartlett's test for testing equality variances of a normal populations.

   b) Let X be a multinomial vector with 4 cells and cell probabilities are $P(c_1) = \theta^2, P(c_2) = P(c_3) = \theta(1-\theta), P(c_4) = (1-\theta)^2$. Obtain the MLE and discuss CAN property of the same. **(7+7)**

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XII)
Multivariate Analysis (New) (CGPA)

Day and Date : Wednesday, 18-11-2015 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

***Instructions** : 1) Attempt **five** questions.*

*2) Q.No. **(1)** and Q.No. **(2)** are **compulsory**.*

*3) Attempt **any three** from Q.No. **(3)** to Q.No. **(7)**.*

*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) Let X be a p×1 random vector such that $X \sim N_p(\mu, \Sigma)$, where rank $(\Sigma) = p$. Which of the following is true ?

a) $E\left[(X-\mu)'\Sigma^{-1}(X-\mu)\right] = 2p,\ V\left[(X-\mu)'\Sigma^{-1}(X-\mu)\right] = 2p$

b) $E\left[(X-\mu)'\Sigma^{-1}(X-\mu)\right] = 2p,\ V\left[(X-\mu)'\Sigma^{-1}(X-\mu)\right] = p$

c) $E\left[(X-\mu)'\Sigma^{-1}(X-\mu)\right] = p,\ V\left[(X-\mu)'\Sigma^{-1}(X-\mu)\right] = p$

d) $E\left[(X-\mu)'\Sigma^{-1}(X-\mu)\right] = p,\ V\left[(X-\mu)'\Sigma^{-1}(X-\mu)\right] = 2p$

2) Let $X_1, X_2, ..., X_n$ be a random sample of size n from p-variate normal distribution with mean vector μ and covariance matrix Σ. The distribution of mean vector $\overline{X}$ is

a) $N_p(\mu, \Sigma)$ b) $N_p(\mu, \frac{1}{n}\Sigma)$ c) $N_p(\mu, \frac{1}{n-1}\Sigma)$ d) $N_p(\frac{1}{n}\mu, \frac{1}{n}\Sigma)$

3) Let $A \sim W_p(m, \Sigma)$ and $a \in R^p$ with $a'\Sigma a \neq 0$. Then distribution of $\frac{a'Aa}{a'\Sigma a}$ is

a) χ^2_p b) χ^2_m c) χ^2_{m-p} d) χ^2_{m-p+1}

4) Canonical correlation is a measure of association between

a) one variable and set of other variables

b) two sets of variables

c) two types of variables

d) none of these

5) Let $X_1, X_2, ..., X_n$ be a random sample of size n from p-variate normal distribution with mean vector 0 and covariance matrix Σ. The MLE of Σ is

a) $\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})(X_i - \overline{X})'$ b) $\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})(X_i - \overline{X})'$

c) $\frac{1}{n}\sum_{i=1}^{n} X_i X_i'$ d) $\sum_{i=1}^{n} X_i X_i'$

B) Fill in the blanks : **5**

1) The first pair of canonical variables have _____ correlation.

2) If X has $N_p(\mu, \Sigma)$ distribution and $Z = \Sigma^{-\frac{1}{2}}(X - \mu)$ then E (Z) = _____

3) Let A has $W_p(n, \Sigma)$ distribution and C is some nonsingular matrix then distribution of CA C′ is ________

4) Hotelling's T^2 is multivariate extension of _____

5) Generalized variance is ________ of covariance matrix.

C) State whether the following statements are **True** or **false** : **4**

1) In factor analysis, original variables are expressed as linear combinations of the factors.

2) Canonical correlation can be negative.

3) If X has $N_p(\mu, \Sigma)$ distribution then all marginal distributions for any subset of X are normally distributed.

4) Wishart matrix is not symmetric.

2. a) Answer the following. **6**

i) Define Hotelling's T^2 and Mahalanobis D^2 statistics.

ii) Obtain the null distribution of Hotelling's T^2 statistic.

b) Write short notes on the following : **8**

i) Rao's U statistic.

ii) Single linkage clustering method.

3. a) Let vector X be distributed according to $N_p(\mu, \Sigma)$, show that the marginal distribution of any set of components of X is multivariate normal with means, variances and covariances obtained by taking proper components of μ and Σ respectively.

b) Let $X \sim N_p(\mu, \Sigma)$. Obtain characteristic function of X. **(8+6)**

4. a) State and prove additive property of Wishart distribution.

b) Define canonical correlations and variates. Show that canonical correlation is a generalization of multiple correlation coefficient. **(6+8)**

5. a) Define principal components. State and prove any two properties of principal components.

   b) Obtain the two principal components and percentage of variation explained by these components if $\Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$. **(7+7)**

6. a) Develop a test for equality of mean vectors of two multivariate normal populations. State your assumptions clearly.

   b) Let $X \sim N_3(\mu, \Sigma)$. Obtain the distribution of $Y = X_1 - X_2 + X_3$. **(8+6)**

7. a) Explain discriminant function. Derive Fisher's best linear discriminant function.

   b) Describe orthogonal factor model with m common factors. **(8+6)**

SLR-MM – 523

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XIII)
Planning and Analysis of Industrial Experiments (New CGPA)

Day and Date : Friday, 20-11-2015 Total Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) Smaller the experimental error ________ efficient the design.
a) less b) more
c) not d) none of the above

2) If AB and BC are confounded with incomplete block in 2^n experiment, then automatically confounded effect is
a) ABC b) AC c) A d) B

3) The degrees of freedom corresponding to error in single replicate design is
a) 0 b) 1
c) 2 d) None of above

4) Confounding is necessary to reduce
a) Block size b) No. of blocks
c) No. of factors d) All of above

5) Fractional factorial experiment reduces
a) factors b) levels of factors
c) both a) and b) d) neither a) nor b)

P.T.O.

B) Fill in the blanks : **5**

1) In factorial experiment one can estimate __________ and __________ effects.

2) The shortest word length in defining relation is called as __________

3) Variables which are hard to control are called __________

4) In 3^3 experiment with factors A, B and C the interaction AB has __________ d.f.

5) Preferably __________ interaction is chosen for confounding.

C) State whether the following statements are **true** or **false** : **4**

1) In 2^3 design, generally we choose ABC as confounding factor.

2) Experimental error is due to experimenter's mistake.

3) For 2^k design the complete model would contain 2^{k-2} effects.

4) In Response Surface Study the factors must be quantitative.

2. a) Define with one example : **6**

i) Minimum aberration design.

ii) Resolution of factorial design.

b) Write short notes on the following : **8**

i) Yates table for 2^3 factorial experiments.

ii) Central Composite Design.

3. a) Describe the random effect model of one-way classification.

b) Describe Taguchi arrays. **(7+7)**

4. a) Explain $\frac{1}{4}$ fraction of 2^k design with suitable example.

b) Write down lay-out of 2^4 confounded design with higher order interaction is confounded. **(8+6)**

5. a) Explain advantages and disadvantages of confounding.

b) Explain partial confounding with illustration. **(7+7)**

6. a) Explain Response Surface methodology.

   b) Define :

      i) Principle fraction

      ii) Aliases sets

      iii) Clearly estimate effects. **(7+7)**

7. a) Explain analysis of 2^n factorial experiment in 'r' replicates.

   b) Describe basic principles of Design of Experiments. **(7+7)**

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SLR-MM – 524

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XIV) (Elective – I)
Time Series Analysis (New CGPA)

Day and Date : Monday, 23-11-2015 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

***Instructions** : 1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) The long term movement of time series is ______________

a) trend b) cyclical variation
c) seasonal variation d) noise

2) If mean and covariance function are both independent of time t, then the process is called ______________

a) Weak stationary b) Strict stationary
c) Evolutionary process d) None of these

3) The ARMA(1,1) process is invertible if ______________

a) $|\theta| > 1$ b) $|\theta| < 1$ c) $|\theta| = 1$ d) $|\theta| > 2$

4) The ______________ data is defined as the original time series data with the estimated seasonal component removed.

a) seasonalised b) seasonal
c) deseasonalised d) none of these

5) For large n, the sample autocorrelations of an iid sequence $Y_1, Y_2, ..., Y_n$ with finite variance are approximately iid with distribution ______________

a) N(0, 1/n) b) N(0, 1) c) N(n, 1/n) d) None of these

B) Fill in the blanks : **5**

1) $\{X_t\}$ is a ______________ stationary time series if $(X_1, ..., X_n)$ is identical in distribution with $(X_{1+h}, ..., X_{n+h})$ for all integers h and $n \geq 1$.

2) An iid sequence is ______________ stationary.

3) A stationary time series is ______________ if $\gamma(h) = 0$ whenever $|h| > q$.

4) A sequence of uncorrelated random variables, each with zero mean and variance σ^2 is called ______________

5) The Spencer 15-point moving average is a filter that passes polynomials upto degree ______________ without distortion.

C) State whether the following statements are **true** or **false** : **4**

1) The random walk is a weak stationary process.
2) Every IID noise is white noise.
3) Every white noise is IID noise.
4) The autocorrelation function $\gamma(h)$ is symmetric in h.

2. a) i) Define Ar(p) Process. Find its Autocorrelation Function (ACF).
ii) Define an invertible process. Give one example. **(3+3)**

b) Write short note on the following :
i) Double exponential smoothing.
ii) Weak and strict stationarity. **(4+4)**

3. a) Define a causal process. State conditions under which an ARMA process is causal. Examine whether the process $X_t + 1.6^*X_{t-1} = Z_t - 0.4^*Z_{t-1}$ is causal.

b) Define MA(q) process. Obtain its autocovariance function. **(7+7)**

4. a) What do you mean by smoothing of a time series ? Also explain Holt-Winter exponential smoothing.

b) Describe the main components of time series. Discuss any one method of trend removal in the absence of a seasonal component. **(6+8)**

5. a) Describe the need of ARCH and GARCH models.

b) Define the ARIMA model. Discuss the problem of forecasting ARIMA models. **(6+8)**

6. a) Describe the test based on turning points for testing randomness of residuals.

b) For the model $(1 - 0.2\ B)\ X_t = (1 - 0.5\ B)\ Z_t$, evaluate the first three π-weights and the first three Ψ-weights. **(6+8)**

7. a) Discuss in brief about Yule-Walker equations.

b) Describe Durbin-Levinson algorithm for fitting AR(p) model. **(6+8)**

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Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XV) (Elective – II)
Regression Analysis (New CGPA)

Day and Date : Thursday, 26-11-2015 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative : **5**

1) The model $y = \theta_1 e^{\theta_2 x} + \in$ is
   a) linear regression model
   b) non-linear regression model
   c) polynomial regression model
   d) none of these

2) The sum of the residuals in any regression model with intercept β_0 is always
   a) positive b) zero
   c) non-zero d) one

3) The variance of i^{th} press residual is
   a) $\frac{\sigma^2}{1-h_{ii}}$ b) σ^2 c) σ^2 (1 – hii) d) $\frac{1-hii}{\sigma^2}$

4) The multicollinearity problem in a multiple linear regression is concern with
   a) the error terms
   b) response variable
   c) the regressors
   d) none of these

5) Coefficient of determination R^2 is defined as

a) $\frac{SS_{Regression}}{SS_{Total}}$ b) $\frac{SS_{residual}}{SS_{Total}}$

c) $1-\frac{SS_{Regression}}{SS_{Total}}$ d) None of these

B) Fill in the blanks : **5**

1) Any model that is linear in the unknown parameters is called ________ regression model.

2) The hat matrix $H = x(x^1x^{-1})x^1$ is symmetric and _________ matrix.

3) __________ test is used to test the significance individual regression coefficient in linear regression model.

4) The regression model $y = \beta_0 + \beta_1 x + \beta_2 x^2$ is called polynomial regression model with _____________ variable(s).

5) Cor $\left(\hat{\beta}\right)$ = ____________, $\hat{\beta}$ is OLS estimator of β.

C) State whether the following statements are **True** or **False** : **4**

1) OLS estimator of regression coefficient is BLUE.

2) Condition indices of matrix $x'x$ is defined as $\frac{\lambda_{max}}{\lambda_{min}} + \lambda j$.

3) Auto correlation is concern with predictor variables.

4) Residuals are useful for detecting outlier observation in x-space.

2. a) Explain the terms :

1) Variance Inflection Factor (VIF)

2) Standardized and studentized residual.

b) Write short notes on the following :

1) Variable selection problem.

2) Box-cox power transformation. **(6+8)**

3. a) Describe multiple linear regression model with K predictor variables. Write model in matrix form and state the basic assumptions. Derive the least square estimator of regression coefficients.

   b) In usual notations, outline the procedure of testing a general linear hypothesis $T\beta = 0$. **(7+7)**

4. a) Describe the problem of multicollinearity with suitable example. What are the effects of the same on least squares estimates of the regression coefficients.

   b) Define Mallows' C_P-Statistic and derive the same. **(7+7)**

5. a) Explain the following terms :

      1) Influential observation

      2) Mallows' class of estimators

      3) Breakdown point.

   b) Define M-estimator and writedown the computational procedure of M-estimator. **(6+8)**

6. a) Describe the least square method for parameter estimation in non-linear regression. Discuss the same for $y = \theta_1 e^{\theta_2 x} + \in$.

   b) Describe Cochrane-Orkut method for parameter estimation in the presence of autocorrelation. **(7+7)**

7. a) Explain :

      i) Kernal regression

      ii) Locally weighted regression.

   b) Discuss Durbin-Watson test for deteching auto correlation. **(7+7)**

SLR-MM – 451

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2015
STATISTICS (Paper – XIX) (CGPA)
Elective – I : Operations Research

Day and Date : Tuesday, 24-11-2015 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No.(**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q.No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative. **5**

1) A necessary and sufficient condition for a b.f.s. to a minimization LPP to be an optimum is that (for all j)

a) $(z_j - c_j) \geq 0$ b) $(z_j - c_j) \leq 0$
c) $(z_j - c_j) = 0$ d) $(z_j - c_j) > 0$ or $(z_j - c_j) < 0$

2) Which of the following is not true ? Dual simplex method is applicable to those LPPs that start with

a) an infeasible solution

b) a feasible solution

c) an infeasible but optimum solution

d) a feasible and optimum solution

3) Consider the LPP

Maximize $Z = 3x_1 + 5x_2$

subject to the constraints,

$x_1 + 2x_2 \leq 4$, $2x_1 + x_2 \geq 6$

and $x_1, x_2 \geq 0$

This problem represents :

a) zero-one IPP b) pure IPP c) mixed IPP d) non-IPP

P.T.O.

4) For a two person game with A and B, the minimizing and maximizing players, the optimum strategies are
   a) minimax for A and maxmin for B
   b) maximax for A and minimax for B
   c) minimin for A and maxmin for B
   d) maximin for A and minimax for B

5) The quadratic form X^TQX is said to be negative semi-definite if
   a) $X^TQX > 0$
   b) $X^TQX < 0$
   c) $X^TQX \geq 0$
   d) $X^TQX \leq 0$

B) Fill in the blanks : **5**

1) A set of vectors $X_1, X_2, \ldots X_n$ which satisfies the constraints of LPP is called _______

2) An optimum solution is considered the __________ among feasible solutions.

3) In dual simplex method the starting basic solution is always_________

4) A Quadratic programming problem is based on __________ simplex method.

5) A pair of strategies (p, q) for which $\underline{V} = \overline{V} = V$ is called _________ of E (p,q).

C) State whether the following statements are **True** or **False.** **4**

1) Linear programming problem is probabilistic in nature.

2) The solution to maximization LPP is not unique if $(z_j - c_j) > 0$ for each of the non- basic variables.

3) Dual simplex method is an alternative method to Big M method.

4) In a two person zero sum game, a game is said to be fair if both the players have equal number of strategies.

2. a) i) Show that dual of the dual of an LPP is primal.
   ii) Explain the graphical method of solving $m \times 2$ game. **6**

b) Write short notes on the following :
   i) Artificial variables
   ii) Unrestricted variables. **8**

3. a) Use two phase method to solve the LPP : **6**

Minimize $Z = x_1 + x_2 + x_3$

$x_1 + x_2 + x_3 + x_4 = 4$

$x_1 + 2x_2 + x_3 + x_4 = 4$

$x_1 + 2x_2 + x_3 = 4$

and $x_1, x_2, x_3, x_4 \geq 0$

b) Use simplex method to solve the problem : **8**

Maximize $Z = 3x_1 + 2x_2 + x_3$

subject to the constraints,

$2x_1 + 5x_2 + x_3 = 12$

$3x_1 + 4x_2 = 11$

$x_1, x_3 \geq 0$, x_2 is unrestricted.

4. a) State and prove basic duality theorem. **8**

b) Obtain an optimum solution, if any, to the following LPP **6**

Maximize $Z = 5x_1 + 8x_2 + 10x_3$

subject to the constraints,

$x_1 + x_2 + 2x_3 \leq 20$

$3x_1 - 2x_2 - x_3 \geq 90$

$2x_1 + 4x_2 + 2x_3 = 100$

and x_1, x_2, x_3 is ≥ 0

5. a) Describe branch and bound method of solving Integer Programming Problem. **7**

b) Solving the following IPP using Gomorey's cutting plane algorithm. **7**

Maximize $Z = 110x_1 + 100x_2$

subject to the constraints,

$6x_1 + 5x_2 \leq 29$

$4x_1 + 14x_2 \leq 48$

and $x_1, x_2 \geq 0$ and integers

6. a) Use Beale's method to solve the following problem. **6**

Minimize $Z = -4x_1 + x_1^2 - 2x_1 x_2 + 2x_2^2$

subject to the constraints,

$2x_1 + x_2 \geq 6$

$x_1 - 4x_2 \geq 0$

and $x_1, x_2 \geq 0$.

b) Describe the Wolfe's method for solving Quadratic Programming Problem. **8**

7. a) Obtain optimal strategies for both players and value of game from the following payoff matrix. **6**

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{cccccc} B_1 & B_2 & B_3 & B_4 & B_5 & B_6 \end{array} \\ \text{Player A} \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 1 & 3 & -1 & 4 & 2 & -5 \\ -3 & 5 & 6 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

b) Explain the following terms : **8**

i) Two person zero sum game.

ii) Pure and mixed strategy.

iii) Principle of dominance

iv) Supporting and separating hyper planes.
